Improved Optimization of The Charge Simulation Method for Sphere- plane Gaps Using Genetic Algorithms

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***Abstract*- A Genetic Algorithm(GA) based approach for accurate modeling of sphere-plane gaps using the point charge simulation method is presented. Optimal arrangement of the simulating charges locations, expressed in spherical coordinates, are automatically arrived at using GA to eliminate dependency on the user experience and judgment. Accurate results, compared with earlier published techniques, are achieved for a wide range of field non-uniformity factor. Recommended assignment factors, based on optimum charge locations, are also derived and presented. The present work shows that the GA can be useful to, automatically, determine the appropriate arrangement of fictitious charges efficiently for accurate field computations.**

I. INTRODUCTION

The principle of the charge simulation method (CSM) is easy to implement computationally and well suited for potential and field solutions of open boundary problems [1- 6]. The method, simply, computes the simulating charge magnitudes by satisfying the boundary conditions at a selected number of contour points along the electrodes surfaces. The unknown charges are computed from the relation (1);

$$[P]\,[Q] = [V] \tag{1}$$

Where [P] is the potential coefficient matrix, [Q] is the column vector of unknown charges; [V] is the column vector of known potentials at the contour points.

As CSM accuracy depends on the type of simulating charge, their numbers and locations, the CSM program for a particular application becomes case specific and depends on experience and judgment of the user. Several optimization methods were, earlier, proposed to deal with these limitations [1-3,6]. Recently, it has been shown, using a GA model, that the CSM becomes less effective for geometries with low non-uniformity factor [1]. However, in this model the charges were confined to the surface of a hypothetical sphere whose radius, the only optimization parameter in this case, is to be determined by the GA. Appropriate arrangements of fictitious charges and contour points are indispensable to obtain accurate solutions[4,5]. Nevertheless, as the charge number becomes larger in 3-D calculations, it is more difficult to obtain the whole charge arrangement both efficiently and accurately. In fact, it may take hours of computational time to reach a reasonable accuracy[2]. It is thus necessary to, further, investigate more efficient automatic arrangement methods of the simulating charges and assess the solution accuracy for various non uniformity values. The geometry of a sphere-plane gap is relevant for systems involving the dynamic of charged particles in electric fields and, for this reason, is the subject of research both theoretically and experimentally[7].

The purpose of study is to employ GAs to, automatically, determine the optimal locations of simulating charges in the CSM; thus eliminating the dependence on the personal judgment and experience of the user in locating the charges through a trial and error procedure. In the present calculations, few point charges, 6 in this case, are used to simulate an electrode system consisting of a sphere above a ground plane in 3-D spherical coordinate system. A wide range for the non-uniformity factor is assumed. A method is proposed to determine an appropriate arrangement, the radial and angular coordinates, of the fictitious simulating charges to achieve a minimum rms error in the potential values on the sphere surface.

II. MODEL DETAILS AND PARAMETERS

Figure (1) shows the geometric model of the sphere-plane gap with the image sphere in 3-D coordinates. The sphere

electrode with radius R, 1 per unit, is placed at a distance h per unit above a grounded conducting plate.

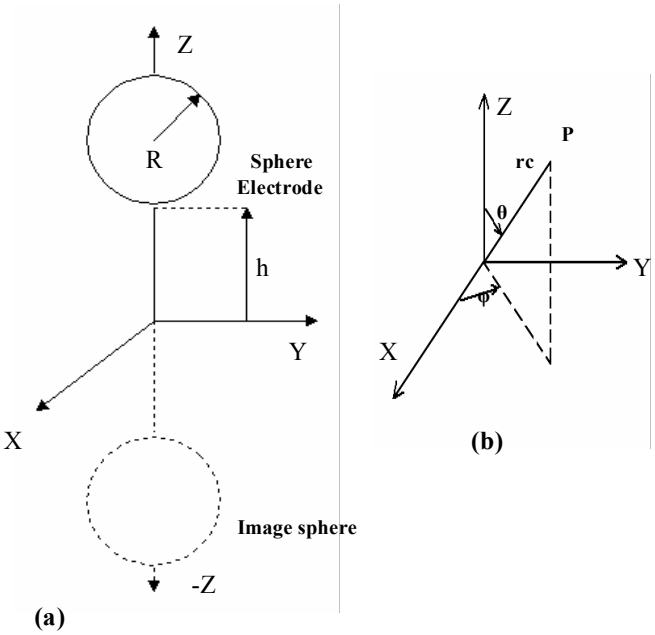

Figure (1) (a) 3-D electrode system, (b) coordinates of a point charge inside the electrode system (r, θ, φ).

A point P inside the sphere electrode can be expressed in spherical coordinate system, whose origin is set at the center of sphere electrode, as P(r, θ, φ) where r is the radial distance from the center of the sphere to point P , θ ϵ [0,π] is the angle between the z axis and the line drawn from the origin to P, and φ ϵ [0,2π] is the angle between the x axis and the line vertically projected on the x-y plane from the point P.

The sphere-plane geometry is simulated using six point charges placed on a hypothetical spherical surface, with radius rc, concentric with the sphere. This radius, rc, can assume values from 0 to R. The θ and φ coordinates of each charge can take any value within the above specified ranges. The contour points are chosen such that, they are on the electrode surface, along the line joining the corresponding charge and the center of the sphere (radial with the charge point location). Earlier[1], these point charges were fixed on the coordinate axes (two on each axis) within the simulating sphere.

Non uniformity factor 'f'' & Assignment factor 'fa'.

The electric field non uniformity factor, *f*, is defined as the maximum electric field intensity in the gap, which occurs at the tip of the high voltage electrode in this case, to the mean electric field intensity (V/h). This factor, largely, depends on the gap spacing h and the sphere radius R. It is usually employed to assess the error variation in simulating the CSM model along with charge and contour point arrangement [8].

The assignment factor 'fa' is defined as the ratio of the distance between a contour point and the corresponding charge point to the distance between two successive contour points. In the present work, the distance between two successive contour points is not a fixed value as earlier assumed [1]. In fact, for each h/R, a range for the assignment factor exists and will have minimum and maximum values (the range of assignment factor). Singer et al. [4] suggested that this factor has a value between 1 and 2. Others [6,7] suggested that this factor lies between 0.7 and 1.5. Generally, the assignment factor does not always lie in the range of 1 and 2. It depends upon the number of simulating charges and decreases as number of simulating charges increases. In actuality, the range over which this parameter can vary is specific to a model based on the type of charges and their number. It is shown here that the range of the optimal assignment factor depends also, for the same model, on the non uniformity of the geometry.

III. GENETIC ALGORITHM (GA)

Recently, GA has been successfully applied in various areas of electric power and high voltage engineering including optimization of charge simulation method [3,6].

GAs manipulate a set of candidate solutions, referred to as a population of individuals and generate a new population at each iteration of the algorithm. Each individual candidate solution is typically represented as a string of decision variables. The variables can be represented in integers or binary digits. The values of the decision variables are manipulated by subjecting the current population of individual strings or chromosomes to a set of standard genetic operators that are inspired by Darwinian evolution theory and are referred to as reproduction, crossover, and mutation. When some optimization criterion is reached (usually based on the number of algorithm iterations or generations), the algorithm terminates. Details of GAs operations are given in [9, 10].

IV. METHOD OF ANALYSIS

For GA calculations, the position of each fictitious charge is described as a ''chromosome''. The r, θ and φ components of a fictitious-charge position are described as sequences of binary digits with lengths of N2 bits. The length of a chromosome is N1*N2bits, where N1 is the number of optimization variables and equals 13 in this case. The first variable is rc (The radius of the hypothetical sphere on which the fictitious charges are placed). The second, θ1, and the third, φ1 variables are the components of the first charge while θ2 and φ2 are those of the second charge and so on. The table shows the structure of a chromosome. Six contour points are placed on the electrode sphere surface radial with the positions of the fictitious charges.

rc	θ_1	Φ_1	θ_2	Φ_2	θ_3	Φ_3	θ_4	Φ_4	θ_5	Φ_5	θ_6	Φ_6

Structure of a chromosome.

The objective function used is the root mean square (RMS) of the potential errors. The RMS value is obtained by evaluating errors at 100 regularly spaced points on the sphere surface. The objective function (fitness function) has the form (2).

$$U = \sqrt{\frac{\sum_{j=1}^{m} [V - \phi(r, \theta, \varphi)]^2}{m}} \qquad (2)$$

Where V is the electrode voltage, ϕ_j is the potential obtained by the CSM and m is the number of check points.

The GA Parameters

Number of fictitious charges of electrode	6	point charges
Number of optimization variables	13	variables
Number of bits per variable per charge	14	bits
Radius of sphere electrode R	1	per unit
Electrode voltage	1	per unit
Population size	4	
Mutation rate	0.02	
Number of generation	10000	

Using roulette wheel selection and single point crossover.

The bit sequences of 4 individuals of the ''0th generation'', the initial condition, are created by using uniformly random numbers. The calculations terminate when the number of generations reach 10000. Based on preliminary tests, the search range for the radius, rc, is between 0.001 to 0.5, while for the angle θ the range is from 0 to π and for the angle φ from 0 to 2π. The sequence of the proposed algorithm is as follows:

1. Determine the domain, for the optimization parameters.
2. GAs generates initial uniform random values for the optimization parameters (rc, θ_1, Φ_1, θ_2 , Φ_2 ,….).
3. For each call to the CSM routine by GAs, the CSM will produce the RMS error for these optimization parameters.
4. The GAs will then evaluate the fitness function and modify the optimization parameters to minimize the RMS error.
5. Steps 3-4 are repeated for a prespecified number of generations (10000 in this case).

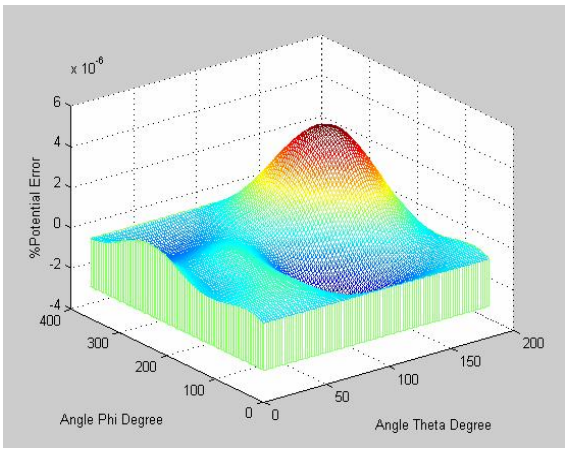

Fig. (2) Plots of potential error(%) for h/R =49.00, f = 49.51

V. RESULTS AND DISCUSSION

An example of the percentage potential error distribution on the sphere surface is given in figure2. The axes are the angle φ and θ coordinates of the points on the sphere. The maximum error in potential in this case is about 4.4e-06 %.

To demonstrate the merits of the proposed method, different gap distances (12 different cases for the non uniformity factor between 1 and 1000) are considered. Figure 3 shows the convergence of the fitness function for these cases. The rms error varies, upon convergence, between 4.6e-4% and 1.3e-6%. The figure shows that the convergence for gap distance 99.500 is relatively faster than the other two gap distances. The results (locations of the charges) for the three cases are given in tables 1-3.

It was observed that as h/R increases, rc generally decreases. While the summation of the magnitudes of the simulation charges also decrease, their values tend to be more uniform for higher h/R. The GA tends to align the charges such that the charge with the largest magnitude usually lies in the lower half of the sphere very close to the z axis with the other charges generally symmetric around the z-axis.

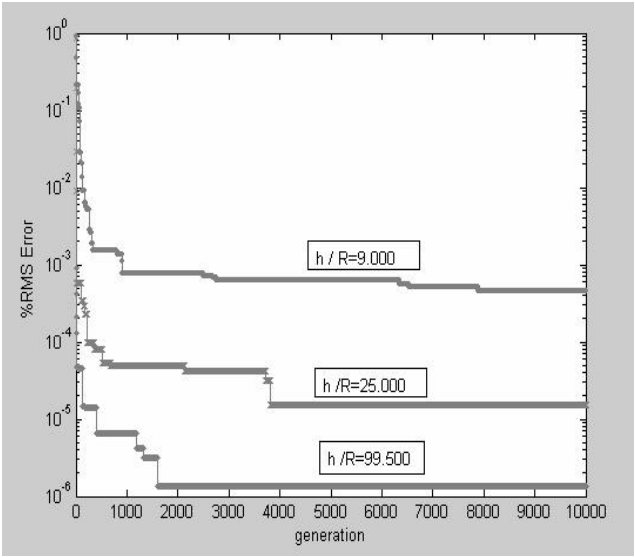

Fig.3 Variation of fitness function with number of generations

Table 1 Locations of the charges for h /R=9.00, f =9.5511

Charges	Magnitudes	Coordinates		
		rc	Theta	Phi
1	0.0067e-10	0.011752	38.6742	151.8403
2	0.2062e-10	0.011752	46.6508	58.7365
3	0.0700e-10	0.011752	38.9929	307.8997
4	-0.3350e-10	0.011752	98.0480	233.5396
5	0.8022e-10	0.011752	168.3648	230.1556
6	0.4205e-10	0.011752	46.7387	230.6611

Table 2 Locations of charges for h /R=25.00, f =25. 5194

Charges	Magnitudes	Coordinates		
		rc	Theta	Phi
1	0.1897e-10	0.00275	14.00842	187.63596
2	0.2596 e-10	0.00275	9.98718	42.10218
3	0.6364 e-10	0.00275	179.53855	107.93628
4	0.0403 e-10	0.00275	43.03607	304.91119
5	-0.0044 e-10	0.00275	147.97290	113.47372
6	0.0122 e-10	0.00275	28.89581	243.69163

Table 3 the locations of charges for h /R=99.50, f =100.00

Charges	Magnitudes	Coordinates		
		rc	Theta	Phi
1	0.2421e-10	0.001041	66.2406	153.1368
2	0.2100e-10	0.001041	96.8724	51.6609
3	0.2814e-10	0.001041	172.1113	249.8444
4	0.0321e-10	0.001041	12.1516	135.9531
5	0.1242e-10	0.001041	94.6420	254.6567
6	0.2278e-10	0.001041	46.1234	316.8650

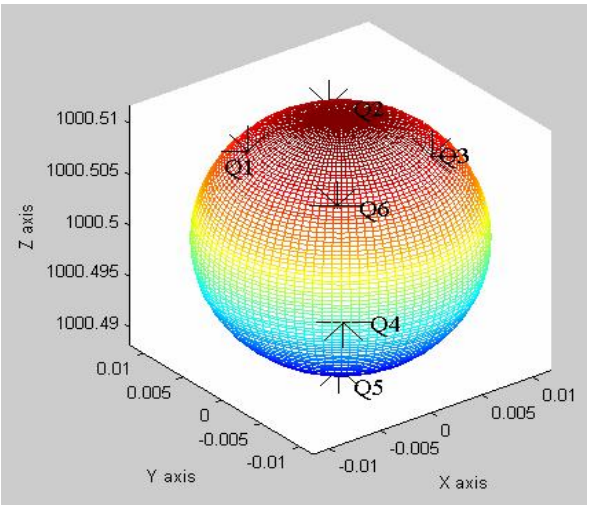

Figure 4 Optimal locations of the simulating charge on the surface of hypothetical sphere at h/R=9.00, f =9.5511.

An example of the optimal locations of the simulating charges (six point charges moving freely on the hypothetical sphere of radius rc) for field non uniformity factor (h/R=9.00, f =9.5511) is shown in figure 4 and given in table 1. This arrangement is generally in conformity with the physical understanding of the geometry of a sphere- plane gap.

The variation of rc for different field non uniformity factors is shown in figure 5. It is seen that this variation is fast for relatively low values of field non uniformity then becomes practically constant at higher values. The optimal values of the locations of fictitious charges for the corresponding field non uniformity factors are obtained (using the GA-CSM program).

The maximum potential error and rms potential error are calculated and shown in figure 6. Plots in figure 6 indicate that, at higher values of field non uniformity factors the simulation errors are the lowest; but higher optimal values are possible only at higher non uniform field factors (as indicated by figure 6 and table 4). This is in general agreement with the results in [1]. However, in [1] all the charges were fixed to the coordinate axes (two on each axis) for all values of non uniformities with the radius of the simulating sphere as the only optimization variable in this case. In the present work, the simulating charges are allowed to move freely on the sphere surface and the optimization parameters in this case are not the sphere radius only, but also the angular position of each charge (θ, φ) for a total number of 13 optimization parameters. These parameters will vary for each non uniformity value.

It is clear that the present approach (moving charges) produce more accurate results than when charges are fixed [1]. These results are reported in table 4 for different values of non uniformity factor and compared with those of ref. [1]. Significant improvement in the accuracy is achieved for the whole range of non uniformity specially for lower values of h/R ($h \leq R$) although errors within this range are still higher than those necessary for accurate field values.

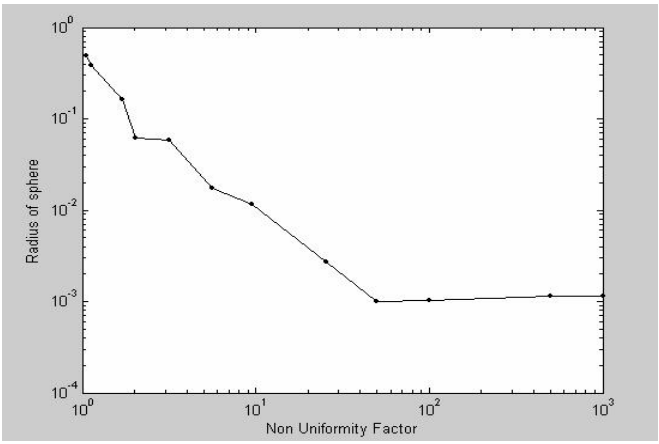

Figure 5Variation of radius rc with f

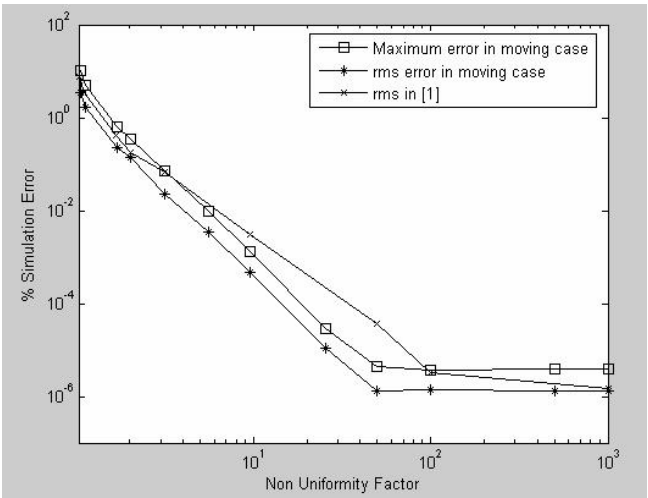

Figure 6 Variation of error (rms and maximum) with f.

The ranges of assignment factors for the different non uniformity factors are shown in table 5. The assignment factor for this method lies between 0.2 - 3.5. All computations have been carried in the MATLAB environment using 1.7 GHZ PC with typical execution time of about 35 minutes including computation time, writing files and printing.

From the range of assignment factor and the value of radius of sphere rc for each value of h /R (non uniformity factor), simple procedures can be developed to obtain appropriate charges locations with reasonable % rms error. With few point charges, 6 in this case and for a given h /R value, place the charges on a sphere surface of radius rc where the minimum and maximum distance between successive contour points lies within the range of assignment factor given in Fig. 7 and Table 5. The figure shows the variation of the hypothetical sphere radius rc and the range of assignment factor for different values of non uniformity factor.

Table 4. RMS errors for different values of h /R

	Present Results		[1]	
h /R	*f*	% rms error	*f*	% rms error
0.1040	1.0527	3.3780e+00	1.0336	7.5446e+00
0.2112	1.1279	1.6749e+00	1.1020	3.4553e+00
0.9197	1.6940	2.2675e-01	1.6846	4.0870e-01
1.3110	2.0283	1.3613e-01	2.0342	1.7520e-01
2.5030	3.1501	2.2992e-02	3.1516	6.9000e-02
9.0000	9.5511	4.6176e-04	9.5511	3.1000e-03
49.0000	49.5100	1.3220e-06	49.5049	3.5794e0-5
99.5000	100.005	1.3623e-06	100.000	3.2093e0-6
999.500	1000.00	1.3189e-06	1000.00	1.4768e0-6

Table 5. Range of assignment factors for different *f*

h /R	*f*	*fa*
0.1040	1.0527	0.1735-0.5408
0.2112	1.1279	0.2441-0.9010
0.9197	1.6940	0.2808-1.5926
1.3110	2.0283	0.4900-1.8300
2.5030	3.1501	0.3312-1.5019
5.0000	5.5860	0.3228-3.4195
9.0000	9.5511	0.3907-1.1186
25.0000	25.5194	0.3365-2.4930
49.0000	49.5100	0.3271-2.8903
99.5000	100.0050	0.3397-1.0470
500.0000	500.5010	0.3599-1.4534
999.5000	1000.0000	0.3599-1.4534

VI. CONCLUSIONS

An improved optimization of the charge simulation method using genetic algorithms has been presented for the geometry

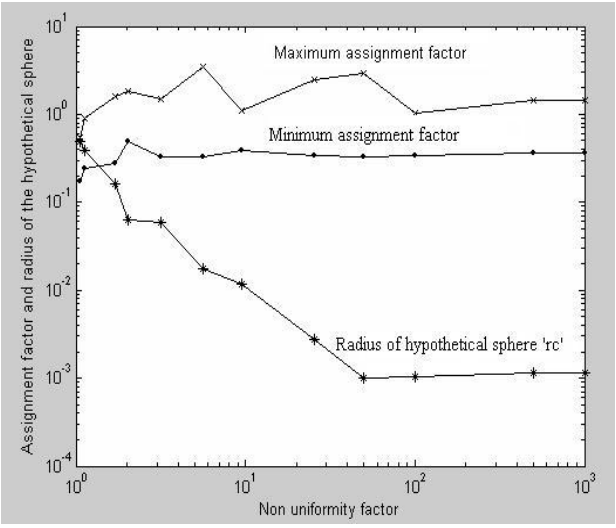

Figure 7 Variation of rc and the range of assignment factor with *f* (different values of h /R).

of a sphere plane gap. The proposed genetic algorithms offers efficiency and accuracy for determination of the optimal locations of simulating charges and their magnitudes. Accurate potential distributions around 3-D spherical electrode system are obtained using a few point charges moving freely on a hypothetical simulating sphere. Accurate results, compared with earlier published techniques, are achieved for a wide range of field non-uniformity factor. Recommended assignment factors, based on optimum charge locations, are also derived and presented. The present work shows that the GA can be useful to, automatically, determine the appropriate arrangement of fictitious charges for accurate field computations.

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